Linear Algebra

(weekly Assignement)

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Submitted Data:

1.Is the trace of square matrice the product of its diagonal entries?justify.Determine whether the set w = { (a1 , a2 , a3)ϵR3 : a1 + 2a2 – 3a3 = 2} is subspace or not of R3 with justification.

**Solution: First part:**

Trace of a square matrix A = aij is define as the sum of elements aii on its principal or leading diagonal.Hence the trace of a square matrix is not the product of its diagonal entries.Square matric have a neat property where the trace of the matrix is equal to the sum of its eigenvalues.

**Second part:**

Here w = { (a1 , a2 , a3)ϵR3 : a1 + 2a2 – 3a3 = 2} we have to show that w is subspace of R3.

For this , we take c1 , c2 ϵR and v1 = (x1 , x2, x3) and v2 = (y1 , y2 , y3) ϵw

So, x1 + 2x2 – 3x3 = -2 ...............(i)

and y1 + 2y2 – 3y3 = -2...................(ii)

since, c1v1 + c2v2

= c1(x1 , x2, x3) + c2 (y1 , y2 , y3)

= (c1x1 +c2y1, c1x2 + c2y2 , c1x3 + c2y3)

Which is a traid form.

Now, by definition of w

(c1x1 + c2y1) + 2(c1x2 + c2y2) – 3(c1x3 + c2y3) = 2

Or,(c1 x1 + 2c1x2 – 3c1x3 ) + (c2y1 + 2c2y2 – 3c2y3) = 2

Or, c1(x1 + 2x2 – 3x3 ) + c2(y1 + 2y2 – 3y3) = 2

Or,c1.2 +c2.2 = 2≠ c1.0 + c2.0 = 0

Or, c1 + c2 = 1 .Hence w is not subspace of R3.

**2.Is** **the zero vector a linear combination of anynonempty set of vectors? Justify. Is the set of all differential real valued functions defined on R a subspace of c(R)? Justify your answer.**

**Solution: First part:**

The zero vector is linear combination of any nonempty set of vector.An empty sum that is, the sum of no vectors, is usually defined to be 0 and with that definition 0 is linear combination of any set of vectors empty or not.

V be a vector space over field F and let (0,0,...,0) be finite set of vectors in V.A linear combination can be written as

C1.0 + c2.0 +. . .+ cn.0 = 0

**Second Part:**

To show differential function is subspace of real valued function define on R a subspace of c(R)

Vss1: Zero is a constant value.We know that differentiation of constant is zero.

Thus, f(x) = 0 for all xϵ R so we have f ϵ c(R)

Vss2:Sum of two differential function is also differential.We also have differential function is

Continuous. Let f(x) ϵ R and f(y) ϵR

Then, f(x) + f(y) ϵ R => f(x) + f(y) ϵ c(R)

Vss3: Scalar multiplication to any differential function is also differential and this implies also

Continuous i.e ᵿ c ϵR and f(x) ϵ R

cf(x) ϵ (R) => cf(x) ϵ c(R).

**3.a Is the span of Ф is Ф ?justify. The vectors u1 = ( 2, -3 , 1) , u2 = (1 , 4 , -2) , u3 = (-8 , 12 , -4) , u4 = ( 1 , 37 , -17) , u5 = (-3 , -5 , 8) generate R3.Find a subset of the set {u1,u2,u3,u4,u5}.**

**Solution: First part:** The empty set has nothing to it thus the smallest linear subspace is zero and hence the span of

empty set is zero.

**Second part:**

The vectors u1 = ( 2, -3 , 1) , u2 = (1 , 4 , -2) , u3 = (-8 , 12 , -4) , u4 = ( 1 , 37 , -17) , u5 = (-3, -5, 8) generate R3 .To find the subset we have to search the linearly dependent set from {u1,u2,u3,u4,u5}.

Here we have {u1 , u2} are linearly independent ,

Since, u3 = -4(u1) and u4 = -3u1 + 7u2

Now , c1u1 + c2u2 + c3u5 = 0

Or, c1(2 , -3 , 1) + c2(1 , 4 , -2) + c3(-3 , -5 , 8) = 0

2c1 + c2 – 3c3 = 0 ...............(i)

-3c1 + 4c2 -5c3 = 0 ..............(ii)

c 1 – 2c2 + 8c3 = 0 ...............(iii)

from (i) and (ii) we get

2c1 + c2 – 3c3 = 0

-2c1 – 4c2 + 16c3 = 0

5c2 – 19c3 = 0 ........(iv)

Again from equation (ii) and (iii) we get

-3c1 + 4c2 – 5c3 = 0

3c1 – 6c2 + 24c3 = 0

-2c2 + 19c3

From (iv) and (v)

3c2 = 0 => c2 = 0 .By solving we get c1 = 0 , c3 = 0

So, {u1 , u2 , u5} be a subset of set {u1 , u2 , u3 , u4 , u5} are linearly independent and

generate R3.So {u1 , u2 , u5} is basis for R3.

**3.b Does every vector space have finite basis? Justify. Prove that the set of solutions to the system of linear equations**

**2x – y + z = 0 3x – 2y + Z = 0 is a subspace of R3.Find the basis for this R3.**

**Solution:**

**First part**

Every vector space have finite basis is false.The space c[0 ,1] or the space of all

Polynomials has no finite basis,only infinite one.

**Second part**

To prove the set of linear equations 2x – y + z = 0 3x – 2y + z = 0 is a subspace of R3.

Now

Let us denote the solution set by S,

Where S = {a(1,1,-1) :a ϵR}

= {(a,a,-a):aϵR} is the solutions set

of equations 2x – y + z = 0 3x – 2y + z = 0.

Now, we show S is the subspace of R3.Clearly S is subspace of R3.

Vss1:Clearly = (0,0,0) ϵ R3 is a zero vector of R3.Now we have to show 0 is zero

vector of S let = (d,d,-d) ϵ s such that

u + 0 = u, ᵿ uϵ s , Where u = (a , a ,-a)

(a , a , -a) +(d , d , -d) = (a , a , -a) => (d,d,-d) = 0.Hence = (0,0,0) is

The solutions to the equcation.Hence = (0,0,0) ϵ S

Vss2: Let u = (a , a ,-a) and v = (b,b,-b) where a,b ϵ R and u , v ϵS

Then u + v = (a , a ,-a) + (b , b , -b)

= (a+b ,a+b , -a-b)

=(c , c , -c) where c = a + b and c ϵ R.Hence u +v ϵS

Vss3:Let u = (a , a ,-a) and k ϵ F

Such that k u = k(a, a , -a) = (ka , kb , -kc)ϵ S .Hence set of solution to the

Given equations is subspace.

2x – y + z = 0

3x – 2y + Z = 0 By solving we get x = z , z = -y .Let x = s , y = s , z = -s we have the

solution would be [ ( s , s , - s) = s ( 1 , 1 , -1) : s ϵ R ] and basis would be (1,1,-1).

**4.Does the zero vector have no basis? Justify . construct the real polynomial degree at most 2 whose graph contaions the point (-4 , 24) ,(1 , 9) and (3 , 3) sketch the graph.**

**Solution:**

**First Part:**

Since o is the only vector in V ,the set s = {0} is the only possible set for a basis. However, s is not a

Linearly independent set since, for example we have a non trivial linear combination 1 .0 = 0 .

Therefore , the subspace v = {0} does not have a basis.

**Second part:**

Here c0 = -4, c1 = 1, c2 = 3 and b0 = 24 , b1 = 9 , b2 = 3 .Here degree is at most 2 ,so n = 2

Now we find,

F0(x) =

=

=

=

F1(x) =

=

=

=

F2(x) =

=

=

Hence the require polynomial is F(x) = = b0F0(x) + *b1f1(x)* + b2f2(x)

F(x) = 24.[ ] +9[] +3.[]

F(x) =

F(x) = -3x +12

**5.Find row echelon form and reduce row echelon form 0 -1 2 3 4**

**2 1 3 4 1**

**A = 1 3 -1 2 3**

**3 3 -6 4 7**

**Solutions:** Here given matrix is 0 -1 2 3 4

2 1 3 4 1

A = 1 3 -1 2 3

3 3 -6 4 7

0 -1 2 3 4

1 3 4 1

A = 1 3 -1 2 3

3 3 -6 4 7 2 is pivot element

1 3 -1 2 3 R1 <->R3

2 1 3 4 1

A1 = 0 -1 2 3 4

3 3 -6 4 7

1 3 -1 2 3

A2 = 0 -5 5 0 -5 R2<- R2 – 2R1

0 -1 2 3 4

0 -6 -3 -2 1 R4 <- R4 – 3R3

1 3 -1 2 3

0 -5 5 0 -5

B = 0 -1 2 3 4

0 -6 -3 -2 1

1 -3 -1 2 3

B1 = 0 1 -1 0 1 R1 <- -1/5R1

0 -1 2 3 4

0 -6 -3 -2 1

1 -3 -1 2 3

0 1 -1 0 1

B2 = 0 0 1 3 5 R2 <- R2 + R1

0 -6 -3 -2 1

1 -3 -1 2 3

0 1 -1 0 1

B3 = 0 0 1 3 5

0 0 -9 -2 7 R3 <- R3 + 6R1

1 -3 -1 2 3

0 1 -1 0 1

C = 0 0 1 3 5

0 0 0 25 52 R2 <- R2 + 9R1

1 -3 -1 2 3

0 1 -1 0 1

0 0 1 3 5

D = 0 0 0 1 52/25 R3 <- 1/25 R3

1 -3 -1 2 3

0 1 -1 0 1

E = 0 0 1 3 5

0 0 0 1 52/25

Matrix E is row echelon form of matrix A.Now reduce row echelon form of A is,

1 0 2 2 0 R1 <-R1 – 3R2

= 0 1 -1 0 1

0 0 1 3 5

0 0 0 1 52/25

1 0 2 2 0

= 0 1 0 4 1 R2 <- R2 +2R1

0 0 1 3 5

0 0 0 1 52/25

1 0 0 -4 -10 R1 <- R1 -2R3

= 0 1 0 4 1

0 0 1 3 5

0 0 0 1 52/25

1 0 0 0 -42/25 R1 <- R1 +4 R4

= 0 1 0 4 1

0 0 1 3 5

0 0 0 1 52/25

1 0 0 0 -42/25 R1 <- R1 +4 R4

= 0 1 0 4 1

0 0 1 3 5

0 0 0 1 52/25

1 0 0 0 -42/25

= 0 1 0 0 193/25 R2 <- R2 – 4R1

0 0 1 0 251/25 R3 <- R3 – 3R1

0 0 0 1 52/25

Which is reduce row echelon form of matrix A